[7] A. H. Compton, *Physic. Rev.*, **21**, 483 (1923).
[8] A. H. Compton, *Ibid.*, **22**, 409 (1923).
[9] Jauncey and Eckart, *Nature*, **112**, 325 (1923).
[10] M. F. Wolfers, *C. R., Paris Acad. Sci.*, **177**, 759 (1923).
[11] E. O. Hulburt, *Phys. Soc. Meeting*, Chicago, 1923.
[12] O. B. Overn, *Physic. Rev.*, **14**, 137 (1919).

REFRACTION OF X-RAYS IN PYRITES

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In all careful determinations of wave-lengths of X-rays by crystal reflection it has been found that the relation

$$n\lambda = 2d \sin \Theta$$

does not hold accurately for the several orders. The departure from this law has been rightly ascribed by Stenstrom[1] to refraction in the crystal. This refraction has been observed by Hjalmar,[2] Davis and Terrill[3] and others, and has been directly confirmed by experiments of Compton on total reflection. If the angles are measured with respect to the crystal surface, the index of refraction μ is expressed by

$$\mu = \frac{\cos \Theta_1}{\cos (\Theta_0 - \varphi)} \qquad (1)$$

where Θ_1 is the angle of incident rays to the surface outside the crystal and φ is the angle of the surface to the molecular planes and Θ_0 is the angle of the X-ray beam to the molecular planes inside the crystal.

Placing $\mu = 1 - \delta$, Stenstrom has derived the following expression for δ, for the case when $\varphi = 0$.

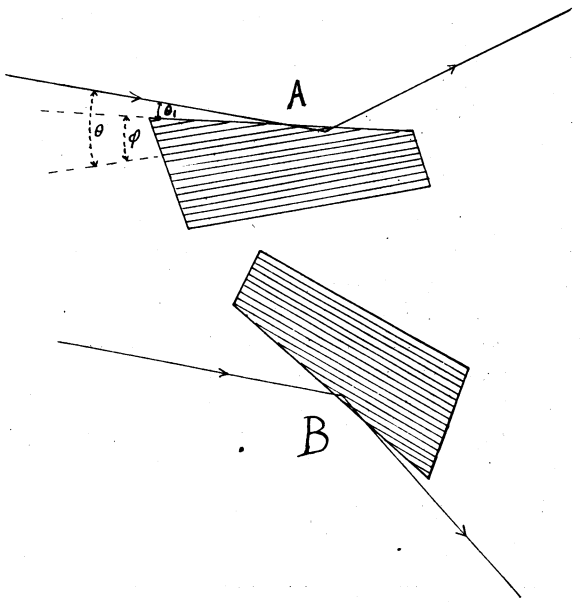
$$\delta = \frac{\left(\frac{\sin \Theta_m}{m}\right)^2 - \left(\frac{\sin \Theta_n}{n}\right)^2}{2\left(\frac{\cos \Theta_m}{m}\right)^2 - 2\left(\frac{\cos \Theta_n}{n}\right)^2}, \qquad (2)$$

where Θ_m, Θ_n are the observed glancing angles at the orders m and n.

The bending of the rays by refraction as they pass through the surface in the case of the natural cleavage surface is small. The bending for Mo, K_{α_1} radiation in calcite for instance is found to be about 3″ arc. Since it is difficult to measure such a small effect accurately, the following method was proposed to increase the bending due to refraction. As in the case

of light, the bending is increased as the angle of the ray to the surface becomes very small. The crystal was consequently cut and optically polished so that the surface made an angle φ to the planes as shown in the figure.

The source of the radiation was a water cooled molybdenum tube kindly supplied by the Research Laboratory of the General Electric Company. The measurements were made with an especially designed ionization spectrometer. This spectrometer is provided with an accurate tangent worm and hand wheel by which small angles can be turned through and read with great accuracy. The worm and hand wheel were calibrated by means of an optical lever device, the distance of the scale from the mirror mounted on the crystal table being about 55 feet.



For the purpose of the accurate measurement of wave-length, it is necessary to turn the crystal through an angle $\beta = (180° - 2$ the glancing angle). This cannot be done with sufficient accuracy by vernier readings. It was accomplished by placing an optically plane-parallel interferometer mirror on the crystal table just above the crystal. A large telescope and brightly illuminated scale were placed at a distance of about 55 feet. Since the mirror was plane-parallel the crystal could be rotated through an angle of 180° with an accuracy of one second. The additional angle could be measured by the optically calibrated tangent worm and hand wheel. The experiments were first performed with the natural (100) face of a very perfect crystal of pyrites. Measurements were made at the 1st

and 4th orders. The value of δ was directly calculated by equation 2. The value so obtained is given in the last column of table 1 for $\varphi = 0$.

The crystal was then ground and polished so that the surface made an angle $\varphi = 6°31'57.5''$ with the reflecting planes. The position of the crystal and path of rays is shown in the figure (A). The crystal is then turned through an angle $\beta = (180-2\alpha)$, at which reflection is then observed. (Position B.) Measurements could be successfully made only at the 1st order for position B, the 4th order being very weak on account of absorption. It may be shown that δ may be expressed in terms of the 1st order readings by the equation

$$\delta = \frac{(\sin\alpha - \sin\Theta_0)(\sin^2\alpha - \sin^2\varphi)}{\sin\alpha\cos^2\alpha}, \qquad (3)$$

where $\alpha = \frac{1}{2}(180° - \text{angle turned through})$ in going from position A to position B, and Θ_0 is the true angle that the rays make with the crystal planes inside the crystal.

The true value of Θ_0 is found by successive approximations as follows: The value of δ obtained for the uncut crystal by equation 2 is used in equation 1 to obtain a tentative value of Θ_0. The tentative Θ_0 is used in equation 3 together with the measured α and φ to calculate a tentative δ. This tentative δ is again used in equation 1 to calculate a more accurate Θ_0. These successive approximations are continued until sufficiently accurate values of Θ_0 and δ are obtained.

The final values of δ and the true value of Θ_0 are given in the table. It will be noticed that the bending of the rays $\Theta_1-(\Theta_0-\varphi)$ on passing through the surface is only 3.6″ for a natural crystal face while it is 39″ for $\varphi = 6° 31'57.5''$ and for $\varphi = 7° 18'39''$, the bending of the ray is much greater being 159″ of arc. The corresponding results for the K_{β_1} line ($\lambda = 0.6311$) are given in the lower part of the table.

TABLE I

$\lambda = 0.7078$

φ	$\alpha = \frac{1}{2}(180°-\beta)$	$\Theta_1-(\Theta_0-\varphi)$	$\delta(\times 10^6)$
0°	7° 31′ 28″	3.6″	4.6
6° 31′ 57.5″	7° 31′ 43.5″	39″	3.26
7° 18′ 39″	7° 32′ 48.5″	159″	3.37
$\Theta_0 = 7° 31' 22.9''$			
$\lambda = .6311$			
0°	6° 42′ 18.5″	3″	3.87
6° 31′ 57.5″	6° 43′ 41″	160″	2.82
$\Theta_0 = 6° 42' 13.4''$			

where φ = angle between crystal surface and planes.
β = angle turned through from position A to B.

θ_1 = angle of ray to surface outside crystal.
θ_0 = true angle of ray to planes inside crystal.

It is of interest to compare these results with the dispersion formula derived for ordinary light by H. A. Lorentz.[4] This formula expressed in the usual electromagnetic units may be written

$$\delta = \frac{e^2}{2\pi m}\left(\frac{n_1}{\nu^2-\nu_1^2}+\frac{n_2}{\nu^2-\nu_2^2}+\text{etc.}\right),$$

where ν is the frequency of the incident radiation and n_1, n_2 are the number of electrons per unit volume that have natural frequencies ν_1, ν_2, etc. The calculated value of δ given in table II was obtained by the above formula.

The weighted means of the experimental results are given in table II. The weighting was controlled by the magnitude of the bending $\theta_1-(\theta_0-\varphi)$ and the accuracy of the measurement of φ.

TABLE II

λ	(EXP.)	(CALC.)
0.6311	2.82×10^{-6}	2.62×10^{-6}
0.7078	3.33×10^{-6}	3.29×10^{-6}

On account of the greater energy in the K_α line, the measurements for it are probably much more accurate than those for the K_β.

The agreement of the experimental results with the theory of Lorentz is important and suggestive. The theory is based primarily on the principle of resonance. Each term becomes very large as the frequency of the incident radiation approaches the natural frequency of any group of electrons. The critical frequencies of the elements investigated up to the present time are too far removed from the incident frequency to determine whether this resonance effect is present or not. It is hoped in the near future to determine the refractive effect of iron pyrites on the K_α radiation of copper. The first term of the Lorentz dispersion formula becomes quite large for this case.

[1] W. Stenstrom. "Untersuchungen der Rontgenspektra." *Dissertation*, Lund, 1919.
[2] Hjalmar, *Zeit. Physik.*, **7**, 1921.
[3] *Proc. Nat. Acad. Sci.*, Dec. 1922.
[4] *Theory of Electrons*, page 159.